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Coverage Probability Analysis of LEO Satellite Communication Systems With Directional Beamforming

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***Abstract*—This paper studies the coverage probability of the low Earth orbit (LEO) satellite satellite communication systems with a directional beamforming. The larger beamwidth of the satellite increases the beam coverage, but the network interference increases. On the other hand, the smaller beamwidth reduces the beam coverage, but the network interference becomes smaller and the desired power can be improved. Thus, the optimal beamwidth control is necessary to maximize the performance. To address it, we model the satellite networks with Poisson point process and analyze the coverage probability as a function of the beamwidth with stochastic geometry. With some numerical examples, we investigate how various system parameters such as altitude and satellite density affect on the optimal beamwidth and demonstrate that the optimal control of the beamwidth of the satellites can maximize the coverage probability.**

***Index Terms*—Satellite systems, coverage probability, stochastic geometry.**

I. Introduction

Recently, the low earth orbit (LEO) satellites have drawn much attention as a promising solution to provide the wireless global connectivity. The spatial modeling of satellite communication networks is of great importance for their design and performance analysis. The conventional model to study the performances of satellite networks is regularly structured deterministic model, where all satellites are evenly spaced and have the same period and inclination [1]. However, this model requires the extensive system level-simulations and makes hard to understand the fundamentals of satellite communication systems with a tractable form.

The stochastic geometry is an useful mathematical tool for studying the performance of the networks and it is widely used to discover the fundamental limits of diverse networks such as cellular network [2], [3] and decentralized network [4], [5]. Recently, there have been also some trials to understand the fundamentals of the satellite communication systems with stochastic geometry [6]–[12]. The coverage and outage probabilities of LEO satellite networks were analyzed based on a binomial point process(BPP) model over Shadowed-Rician fading channel [6], [7]. However, these works did not consider the network interferences. The coverage probability and the average rate of LEO satellite networks were investigated based on BPP over Rayleigh fading [8]. The coverage probability was studied with a Poisson point process(PPP) over Nakagami-m fading channel, but the beamwidth control issue was not studied [9]. The coverage probability and average data rate of LEO network were analyzed with a non-homogeneous PPP model [10] and the optimal altitude of PPP based dense satellite constellations to maximize the coverage probability was investigated [11]. The algorithm to calculate distance of different point sets was proposed and the distance between BPP and Fibonacci lattice/orbit models was compared [12]. Obviously, none of the above works have successfully addressed the beamwidth control of directional beamforming, which is the main focus of this paper.

When the satellites perform a directional beamforming, the larger beamwidth of the satellite increases the beam coverage, but the network interference increases. On the other hand, the smaller beamwidth reduces the beam coverage, but the network interference becomes smaller and the desired power can be improved. Therefore, the optimal beamwidth control is required to properly balance the network interference and the beam coverage. To address it, we analyze the coverage probability of the satellite communication networks with a stochastic geometry and investigates the optimal beamwidth. With numerical examples, we demonstrate that the optimal control of the beamwidth of the satellites can maximize the coverage probability and investigate how various system parameters such as channel parameters and network topology affect on the coverage probability.

II. System Model

We consider the downlink satellite communication systems performing a directional beamforming, where the satellites are uniformly distributed over the spherical surface of altitude H [km] from the Earth, which is depicted in Fig. 1. The Earth is assumed as a sphere of a radius $R_e(\sim 6{,}371$ [km]) centered at the origin $\mathbf{o} \triangleq (0,0,0) \in \mathbb{R}^3$ in the 3D Cartesian coordinate system. We model the satellites as a homogeneous PPP with

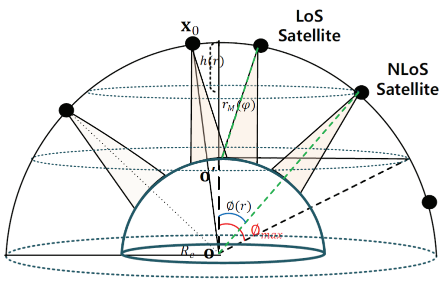

Fig. 1: System model

intensity λ and denote their location set as $\Phi = \{\mathbf{x}_l\}$, where $\mathbf{x}_l \in \mathbb{R}^3$ indicates the location of l-th satellite.

We focus on the downlink performance of a reference user which is located at the position $\mathbf{o}' \triangleq (0, 0, R_e)$ on the surface of the Earth and equipped with an omni-directional single antenna of which gain is normalized to one. We define the surface area of spherical dome (cap) above the horizon of the reference user as the *satellite-visible region* and denote it as S_{V}. Note that the reference user can only see the satellites in the satellite-visible region. The user is associated with the nearest satellite among the satellites covering the user with the beam in the satellite-visible region. If there is no satellite that can serve the user with the beam, the user cannot be served from the satellites. Let us denote the location set of satellites in the satellite-visible region as Φ_{V} and the set of satellites which can cover the user with the beam (referred to *beam coverage satellites*) as $\Phi_{\mathrm{M}}(\subset \Phi_{\mathrm{V}})$.

We denote the location of the serving satellite as $\mathbf{x}_0 \in \mathbb{R}^3$. Then, the distance from the reference user to the serving satellite can be represented as $R_0 = \|\mathbf{x}_0 - \mathbf{o}'\|$, where $H \leq R_0 \leq r_{\mathrm{M}}(\varphi)$, where $r_{\mathrm{M}}(\varphi)$ is the maximum distance from the user to the serving satellite. $r_{\mathrm{M}}(\varphi)$ can be represented by $r_{\mathrm{M}}(\varphi) = (R_e+H)\cos\frac{\varphi}{2} - \sqrt{(R_e+H)^2\cos^2\frac{\varphi}{2} - (2R_eH + H^2)}$. This solution is obtained by solving the cosine rule equation $R_e^2 = r_{\mathrm{M}}^2(\varphi) + (R_e+H)^2 - 2r_{\mathrm{M}}(\varphi)(R_e+H)\cos(\varphi/2)$. We note that $r_{\mathrm{M}}(\varphi)$ increases as φ increases. For a given distance $R_0 = r$, the surface area of spherical cap can be represented by $S(r) = \pi(R_e+H)(r^2 - H^2)/R_e$ and the zenith angle in Fig. 1 can be represented by $\phi(r) = \arccos\left(1 - \frac{r^2 - H^2}{2R_e(R_e+H)}\right)$. The number of satellites in $S(r)$ follows the Poisson distribution, so it can be represented from the void probability as $\mathbb{P}[N(S(r)) = k] = \frac{(\lambda S(r))^k}{k!}e^{-\lambda S(r)}$, where $k = \{0, 1, \cdots, \infty\}$.

The satellites perform a directional beamforming which radiates an ideal conical beam of beamwidth $\varphi(>0)$ towards the center of the Earth. Thus, the transmit antenna beam gain can be represented as

$$G(\varphi) = \begin{cases} \min\left\{G_{\max}, \frac{1-\cos(\varphi_{\max}/2)}{1-\cos(\varphi/2)}\right\} & \text{for } |\varphi| \leq \varphi_{\max} \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where $\varphi_{\max} = \arccos\left(\frac{H^2 + 2HR_e - R_e^2}{(R_e+H)^2}\right)$ represents the maximum beamwidth to cover the Earth and $G_{\max}$ represents the maximum beam gain. Note that the antenna gain is normalized as one when $\varphi = \varphi_{\max}$. As the beamwidth decreases, the transmit antenna gain increases but is saturated to $G_{\max}$.

We consider a simplified LoS/NLoS transmission model, where LoS and NLoS transmission occurs with pathloss exponent α_{L} and α_{N} when $R \leq r_{\mathrm{LN}}$ and $R > r_{\mathrm{LN}}$, respectively. Then, the pathloss gain can be expressed as

$$L(R) = \begin{cases} L_0 R^{-\alpha_{\mathrm{L}}}, & \text{for } R \leq r_{\mathrm{LN}}, \\ L_0 R^{-\alpha_{\mathrm{N}}}, & \text{for } R > r_{\mathrm{LN}}, \end{cases} \quad (2)$$

or, equivalently as

$$L(R) = \begin{cases} L_0 R^{-\alpha_{\mathrm{L}}}, & \text{for } \phi(R) \leq \phi_{\mathrm{LN}}, \\ L_0 R^{-\alpha_{\mathrm{N}}}, & \text{for } \phi(R) > \phi_{\mathrm{LN}}, \end{cases} \quad (3)$$

where $\phi_{\mathrm{LN}} \triangleq \phi(r_{\mathrm{LN}}) = \arccos\left(\frac{R_e^2 + (R_e+H)^2 - r_{\mathrm{LN}}^2}{2R_e(R_e+H)}\right)$ and $L_0 = (c/(4\pi f_c))^2$, where c is the speed of light and f_c is the carrier frequency. Note that ϕ_{LN} can be any value within $[0, \phi_{\max}(= \arccos(R_e/(R_e+H)))]$ (rad) and it is determined by the surrounding environment of the user such as a big mountain. The satellites in the satellite-visible region can be categorized into two groups: LoS and NLoS satellites. Let us denote the location sets of LoS and NLoS satellites in the satellite-visible region as Φ_{L} and $\Phi_{\mathrm{N}} = \Phi_{\mathrm{V}} \setminus \Phi_{\mathrm{L}}$, respectively. We also denote the location sets of beam coverage satellites serving the user via LoS and NLoS transmission as $\Phi_{\mathrm{M,L}}$ and $\Phi_{\mathrm{M,N}}$, respectively, where $\Phi_{\mathrm{M}} = \Phi_{\mathrm{M,L}} \cup \Phi_{\mathrm{M,N}}$.

We consider the Nakagami-m fading as the small scale fading channel model for the satellites [9], [10]. The small-scale fading channels for LoS and NLoS links follow the independent Nakagami-m fading distributions with different fading parameters m_{L} and m_{N}. The channel power gain distribution is represented as

$$f_{|h|^2}(x) = \frac{m_\zeta^{m_\zeta}}{\Gamma(m_\zeta)} x^{m_\zeta - 1} e^{-m_\zeta x}, \; x \geq 0, \; \zeta = \{\mathrm{L}, \mathrm{N}\}, \quad (4)$$

where $\Gamma(t) = \int_0^\infty x^{t-1}e^{-x}dx$ is the gamma function. We assume that both m_{L} and m_{N} are positive integers for analytical tractability [13].

When the user is associated with the nearest satellite among the beam coverage satellites, its received signal-to-interference-plus-noise ratio (SINR) can be expressed as

$$\mathrm{SINR} = \frac{G(\varphi)P|h_0|^2 L(R_0)}{N_0 W + I}, \quad (5)$$

where W is the system bandwidth, N_0 is the noise power spectral density, and I represents the aggregate interference given by

$$I = \sum_{l>0:\mathbf{x}_l \in \Phi_{\mathrm{M}} \setminus \mathbf{x}_0} G(\varphi)P|h_l|^2 L(R_l), \quad (6)$$

where $|h_0|^2$ and $|h_l|^2$ represent the channel power gains of desired link and l-th interfering link, respectively, and $R_l = \|\mathbf{x}_l - \mathbf{o}'\|$ represent the distance to l-th interfering satellite located at $\mathbf{x}_l$.

III. Coverage Probability Analysis

In this section, we analyze the coverage probability of the reference user. Recall that the typical user can be served from the satellite only if there exists at least one satellite in the satellite-visible region and the typical user is within the beam coverage of satellite. Therefore, let us denote the event that there exists at least one satellite in the satellite-visible region as $\mathtt{E}_{\mathrm{V}}$ and the event that the typical user is located within the beam coverage of the closest satellite as $\mathtt{E}_{\mathrm{M}}$. Then, the coverage probability which is defined as the probability that the received SINR is larger than a pre-determined target SINR threshold can be expressed as

$$P_c(\tau) = \mathbb{P}\left[\mathrm{SINR} \geq \tau | \mathtt{E}_{\mathrm{V}}, \mathtt{E}_{\mathrm{M}}\right] \mathbb{P}\left[\mathtt{E}_{\mathrm{V}}, \mathtt{E}_{\mathrm{M}}\right]. \tag{7}$$

Since the typical user can be served only if there exists at least one satellite in $S(r_{\mathrm{M}}(\varphi))$, $\mathbb{P}\left[\mathtt{E}_{\mathrm{V}}, \mathtt{E}_{\mathrm{M}}\right]$ can be represented as $P_{\mathrm{V,M}} = 1 - e^{-\lambda\pi K(r_{\mathrm{M}}^2(\varphi) - H^2)}$, where $K = (R_e + H)/R_e$.

A. Association Probability & Statistical Distance Distribution

We derive some statistics required to analyze the coverage probability. Depending on the relationship between r_{LN} and $r_{\mathrm{M}}(\varphi)$, we can consider two different scenarios: 1) $r_{\mathrm{LN}} \leq r_{\mathrm{M}}(\varphi)$ and 2) $r_{\mathrm{LN}} > r_{\mathrm{M}}(\varphi)$.

1) Scenario 1: $r_{\mathrm{M}}(\varphi) \geq r_{\mathrm{LN}}$: Conditioned on the events $\mathtt{E}_{\mathrm{V}}$ and $\mathtt{E}_{\mathrm{M}}$, the association probabilities to the LoS satellite and NLoS satellite are given as the following lemma.

Lemma 1: When $r_{\mathrm{M}}(\varphi) \geq r_{\mathrm{LN}}$, conditioned on that there exists at least one satellite in the satellite-visible region and the user is located within the satellite beam coverage, the probability to be associated with the LoS satellite is given by

$$A_{\mathrm{M,L}} = \frac{1 - e^{-\lambda\pi K(r_{\mathrm{LN}}^2 - H^2)}}{1 - e^{-\lambda\pi K(r_{\mathrm{M}}^2(\varphi) - H^2)}}, \tag{8}$$

and that with the NLoS satellite is given by

$$A_{\mathrm{M,N}} = \frac{e^{-\lambda\pi K(r_{\mathrm{LN}}^2 - H^2)} - e^{-\lambda\pi K(r_{\mathrm{M}}^2(\varphi) - H^2)}}{1 - e^{-\lambda\pi K(r_{\mathrm{M}}^2(\varphi) - H^2)}}. \tag{9}$$

Proof: We omit the proof due to the limited space. □

Lemma 2: When $r_{\mathrm{M}}(\varphi) \geq r_{\mathrm{LN}}$, conditioned on that there exists at least one satellite in satellite-visible region and the user is located within the beam coverage of LoS satellite, the distance distribution to the serving satellite is given by

$$f_{R_0}^{\mathrm{M,L}}(r) = \frac{1}{A_{\mathrm{M,L}}} \frac{2\lambda\pi K r e^{-\lambda\pi K\left(r^2 - H^2\right)}}{1 - e^{-\lambda\pi K(r_{\mathrm{M}}^2(\varphi) - H^2)}}, \tag{10}$$

where $K = (R_e + H)/R_e$ and $H \leq r \leq r_{\mathrm{LN}}$. Similarly, conditioned on that there exists at least one satellite in satellite-visible region and the typical user is located within the beam coverage of NLoS satellite, the distance distribution to the serving satellite is given by

$$f_{R_0}^{\mathrm{M,N}}(r) = \frac{1}{A_{\mathrm{M,N}}} \frac{2\lambda\pi K r e^{-\lambda\pi K\left(r^2 - H^2\right)}}{1 - e^{-\lambda\pi K(r_{\mathrm{M}}^2(\varphi) - H^2)}}, \tag{11}$$

where $r_{\mathrm{LN}} \leq r \leq r_{\mathrm{M}}(\varphi)$.

Proof: We omit the proof due to the limited space. □

2) Scenario 2: $r_{\mathrm{M}}(\varphi) < r_{\mathrm{LN}}$: When $r_{\mathrm{M}}(\varphi) < r_{\mathrm{LN}}$, the typical user is always located within the beam coverage of LoS satellite only.

Lemma 3: When $r_{\mathrm{M}}(\varphi) < r_{\mathrm{LN}}$, conditioned on that there exists at least one satellite in the satellite-visible region and the typical user is located within the beam coverage of the satellite, the probability to be associated with the LoS satellite is given by $A_{\mathrm{M,L}} = 1$ and that with NLoS satellite is given by $A_{\mathrm{M,N}} = 0$.

Proof: The proof of this lemma is trivial. □

Lemma 4: When $r_{\mathrm{M}}(\varphi) < r_{\mathrm{LN}}$, conditioned on that there exists at least one satellite in satellite-visible region and the typical user is located within the beam coverage of the LoS satellite, the distance distribution to the serving satellite is given by

$$g_{R_0}^{\mathrm{M,L}}(r) = \frac{2\lambda\pi K r e^{-\lambda\pi K\left(r^2 - H^2\right)}}{1 - e^{-\lambda\pi K(r_{\mathrm{M}}^2(\varphi) - H^2)}}, \tag{12}$$

where $K = (R_e + H)/R_e$ and $H \leq r \leq r_{\mathrm{M}}(\varphi)$.

Proof: We omit the proof due to the limited space. □

B. Analysis of Coverage Probability

Let us define the events that the user is served from the LoS satellite and NLoS satellite as $\mathtt{E}_{\mathrm{L}}$ and $\mathtt{E}_{\mathrm{N}}$, respectively. Then, the coverage probability in (7) can be re-written by

$$\begin{aligned} P_c(\tau) &= \left(\mathbb{P}\left[\mathrm{SINR} \geq \tau | \mathtt{E}_{\mathrm{L}}, \mathtt{E}_{\mathrm{V}}, \mathtt{E}_{\mathrm{M}}\right] \mathbb{P}\left[\mathtt{E}_{\mathrm{L}} | \mathtt{E}_{\mathrm{V}}, \mathtt{E}_{\mathrm{M}}\right] \right. \\ &\quad \left. + \mathbb{P}\left[\mathrm{SINR} \geq \tau | \mathtt{E}_{\mathrm{N}}, \mathtt{E}_{\mathrm{V}}, \mathtt{E}_{\mathrm{M}}\right] \mathbb{P}\left[\mathtt{E}_{\mathrm{N}} | \mathtt{E}_{\mathrm{V}}, \mathtt{E}_{\mathrm{M}}\right]\right) P_{\mathrm{V,M}} \quad (13) \\ &= \left(P_c^{\mathrm{M,L}}(\tau) A_{\mathrm{M,L}} + P_c^{\mathrm{M,N}}(\tau) A_{\mathrm{M,N}}\right) P_{\mathrm{V,M}}. \quad (14) \end{aligned}$$

1) Scenario 1: $r_{\mathrm{M}}(\varphi) \geq r_{\mathrm{LN}}$: If $H \leq r \leq r_{\mathrm{LN}}$, then the reference user can be served from the LoS satellite and experiences both LoS and NLoS interferences. If $r_{\mathrm{LN}} < r \leq r_{\mathrm{M}}(\varphi)$, then the user can be served from the NLoS satellite and experiences only NLoS interferences.

When $H \leq r \leq r_{\mathrm{LN}}$, the typical user is associated with the LoS satellite and its received SINR is given by

$$\mathrm{SINR} = \frac{G(\varphi) P |h_0|^2 L_0 R_0^{-\alpha_{\mathrm{L}}}}{I_1}, \tag{15}$$

where $I_1 = N_0 W + \sum_{\mathbf{x}_l \in \Phi_{\mathrm{M,L}} \setminus \mathbf{x}_0} G(\varphi) P |h_l|^2 L_0 R_l^{-\alpha_{\mathrm{L}}} + \sum_{\mathbf{x}_l \in \Phi_{\mathrm{M,N}}} G(\varphi) P |h_l|^2 L_0 R_l^{-\alpha_{\mathrm{N}}}$. When $r_{\mathrm{LN}} < r \leq r_{\mathrm{M}}(\varphi)$, the typical user can be served from the NLoS satellite and its received SINR can be expressed as

$$\mathrm{SINR} = \frac{G(\varphi) P |h_0|^2 L_0 R_0^{-\alpha_{\mathrm{N}}}}{I_2}, \tag{16}$$

where $I_2 = N_0 W + \sum_{\mathbf{x}_l \in \Phi_{\mathrm{M,N}} \setminus \mathbf{x}_0} G(\varphi) P |h_l|^2 L_0 R_l^{-\alpha_{\mathrm{N}}}$.

Theorem 1: When the typical user is associated with the nearest satellite among the beam coverage satellites, the coverage probability is given by

$$P_c(\tau) = P_{\mathrm{V,M}} \left(P_c^{\mathrm{M,L}}(\tau) A_{\mathrm{M,L}} + P_c^{\mathrm{M,N}}(\tau) A_{\mathrm{M,N}}\right), \tag{17}$$

where

$$P_c^{\mathrm{M,L}}(\tau) = \int_H^{r_{\mathrm{LN}}} \sum_{k=0}^{m_{\mathrm{L}}-1} \frac{(-s)^k}{k!} \frac{d^k}{ds^k} \mathcal{L}_{I_1}(s|r) \Big|_{s=s_{\mathrm{L}}} f_{R_0}^{\mathrm{M,L}}(r) dr, \tag{18}$$

$$P_c^{\mathrm{M,N}}(\tau)=\int_{r_{\mathrm{LN}}}^{r_{\mathrm{M}}(\varphi)}\sum_{k=0}^{m_{\mathrm{N}}-1}\frac{(-s)^k}{k!}\frac{d^k}{ds^k}\mathcal{L}_{I_2}(s|r)\Big|_{s=s_{\mathrm{N}}} f_{R_0}^{\mathrm{M,N}}(r)dr, \tag{19}$$

where

$$\mathcal{L}_{I_1}(s|r)=e^{-sN_0W-\lambda 2\pi(R_e+H)^2\left\{\int_{\phi(r)}^{\phi_{\mathrm{LN}}}\psi_{\mathrm{L}}(\phi)\sin\phi d\phi\right\}} \times e^{-\lambda 2\pi(R_e+H)^2\left\{\int_{\phi_{\mathrm{LN}}}^{\phi(r_{\mathrm{M}}(\varphi))}\psi_{\mathrm{N}}(\phi)\sin\phi d\phi\right\}}, \tag{20}$$

$$\mathcal{L}_{I_2}(s|r)=e^{-sN_0W-\lambda 2\pi(R_e+H)^2\left\{\int_{\phi(r)}^{\phi(r_{\mathrm{M}}(\varphi))}\psi_{\mathrm{N}}(\phi)\sin\phi d\phi\right\}}, \tag{21}$$

where $\psi_\zeta(\phi) = 1-(m_\zeta/(sPL_0G(\varphi)v(\phi)^{-\alpha_\zeta}+m_\zeta))^{m_\zeta}$ and $s_\zeta = m_\zeta\tau r^{\alpha_\zeta}(G(\varphi)PL_0)^{-1}$ for $\zeta\in\{\mathrm{L},\mathrm{N}\}$, $\phi(r_{\mathrm{M}}(\varphi)) = \arccos\left(1-\frac{(r_{\mathrm{M}}(\varphi))^2-H^2}{2R_e(R_e+H)}\right)$, and $v(\phi) = \sqrt{R_e^2+(R_e+H)^2-2R_e(R_e+H)\cos\phi}$.

Proof: We omit the proof due to the limited space. □

Note that as φ increases, the first product term in (17) increases due to the improved beam coverage. However, the integration range of the conditional Laplace transform of aggregated interference plus noise also increases. This implies that more larger number of satellites interferes with the reference user. The opposite result occurs if φ decreases. Therefore, the optimal control of beamwidth is necessary to efficiently balance the network interference and the beam coverage. Unfortunately, (17) is non-convex function with the very complicated form, the optimal beamwidth can be found by relying on the brute-force searching.

2) Scenario 2: $r_{\mathrm{M}}(\varphi) < r_{\mathrm{LN}}$: When $H\le r\le r_{\mathrm{M}}(\varphi)$, the typical user can be served from the LoS satellite and its received SINR can be expressed as

$$\mathrm{SINR}=\frac{G(\varphi)P|h_0|^2L_0R_0^{-\alpha_{\mathrm{L}}}}{I_3}, \tag{22}$$

where $I_3 = N_0W+\sum_{\mathbf{x}_l\in\Phi_{\mathrm{M,L}}\setminus\mathbf{x}_0}G(\varphi)P|h_l|^2L_0R_l^{-\alpha_{\mathrm{L}}}$.

Theorem 2: When the typical user is associated with the nearest satellite among the beam coverage satellites, the coverage probability is given by

$$P_c(\tau)=P_{\mathrm{V,M}}\cdot P_c^{\mathrm{M,L}}(\tau), \tag{23}$$

where

$$P_c^{\mathrm{M,L}}(\tau)=\int_{H}^{r_{\mathrm{M}}(\varphi)}\sum_{k=0}^{m_{\mathrm{L}}-1}\frac{(-s)^k}{k!}\frac{d^k}{ds^k}\mathcal{L}_{I_3}(s|r)\Big|_{s=s_{\mathrm{L}}} g_{R_0}^{\mathrm{M,L}}(r)\,dr, \tag{24}$$

$$\mathcal{L}_{I_3}(s|r)=e^{-sN_0W-\lambda 2\pi(R_e+H)^2\left\{\int_{\phi(r)}^{\phi(r_{\mathrm{M}}(\varphi))}\psi_{\mathrm{L}}(\phi)\sin\phi d\phi\right\}}, \tag{25}$$

where $\psi_{\mathrm{L}}(\phi) = 1-(m_{\mathrm{L}}/(sPL_0G(\varphi)v(\phi)^{-\alpha_{\mathrm{L}}}+m_{\mathrm{L}}))^{m_{\mathrm{L}}}$, $\phi(r_{\mathrm{M}}(\varphi))=\arccos\left(1-\frac{(r_{\mathrm{M}}(\varphi))^2-H^2}{2R_e(R_e+H)}\right)$, and $g_{R_0}^{\mathrm{M,L}}(r)$ is given in (12).

Proof: We omit the proof due to the limited space. □

IV. Numerical Results

In this section, we provide some numerical examples to verify our analytical results in the previous Section and provide some useful design insights. We investigate how various system parameters such as Nakagami-m fading parameters

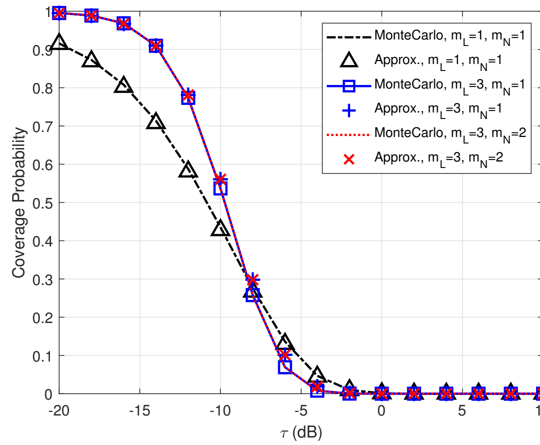

Fig. 2: Comparison of the coverage probabilities versus τ (dB) for various m_{L} and m_{N}.

and satellite density affect the performance and the optimal beamwidth. Unless otherwise stated, the baseline simulation environment is set as follows: $P = 45$ (dBm), $H = 600$ (km), $\lambda = 3.3\times10^{-6}$ (units/km^2), N_0 = -174 (dBm), $f_c = 2$ (GHz), $W = 20$ (MHz), $r_{\mathrm{LN}} = 1,000$ (km), $\alpha_{\mathrm{L}} = 2$, $\alpha_{\mathrm{N}} = 2.3$, $m_{\mathrm{L}} = 3$, and $m_{\mathrm{N}} = 2$, $\varphi = \frac{2}{3}\pi$. In this setting, the average number of satellites over the surface of Earth is nearly 2,000 and $r_{\mathrm{M}}(\varphi) = 1,450$ (km).

Fig. 2 compares the coverage probability in (17) with its Monte-Carlo simulation versus τ (dB) for various m_{L} and m_{N}. This figure verifies that our analytical results are well matched to Monte-Carlo simulation results. Fig. 2 shows that the coverage probabilities when $(m_{\mathrm{L}}, m_{\mathrm{N}})$ = (3,1) and $(m_{\mathrm{L}}, m_{\mathrm{N}})$ = (3,2) are almost the same, while they have some gaps when $(m_{\mathrm{L}}, m_{\mathrm{N}})$ = (3,1) and $(m_{\mathrm{L}}, m_{\mathrm{N}})$ = (1,1). This implies that the fading parameter for LoS link dominantly affect the coverage probability compared to that for NLoS link.

Fig. 3 compares the coverage probabilities versus φ (rad) for various λ (units/km^2). Fig. 3 validates that the optimal control of the beamwidth for λ maximizes the coverage probability. Interestingly, the optimal beamwidth decreases as λ increases. This is because as λ increases, the beamwidth coverage is sufficiently high enough even with narrow beamwidth and thus reducing the interference with the narrow beamwidth is more beneficial. On the other hand, for smaller λ, the beamdwidth should be more larger to guarantee the beamwidth coverage event.

Fig. 4 compares the coverage probabilities between PPP model and deterministic model [12] versus τ (dB) for various λ (units/km^2). The point set of deterministic model is regularly structured with the same distance, while the point set of the PPP model is uniformly distributed. Obviously, the coverage probability of PPP model is lowerbound compared to that of deterministic model. As λ increases, the performance gap between two models becomes negligible. For small λ, there exists a large gap between two models for small τ. This is because the probability of beam contact in PPP model

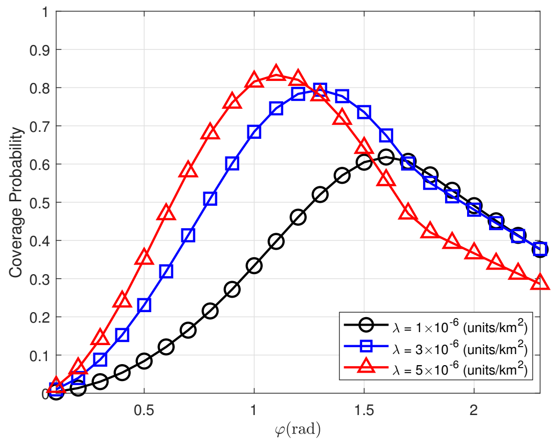

Fig. 3: Comparison of the coverage probabilities versus φ (rad) for various λ.

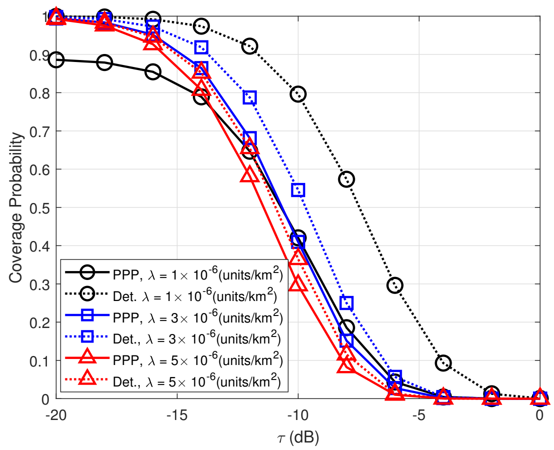

Fig. 4: Comparison of the coverage probabilities between Deterministic model vs PPP model.

becomes more smaller than that in deterministic model for relatively small number of satellites.

V. Conclusion

We have studied the coverage probabilities of LEO satellite communication systems with a directional beamforming with stochastic geometry. Our analytical framework have provided some useful design insights for how to optimally control the beamwidth to maximize the performance of the LEO satellite systems. We have demonstrated that the optimal control of beamwidth can maximize the performances and investigated how various system parameters, such as satellite density and altitude affect the optimal beamwidth.

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